Analysis of Normal and Superconducting Coplanar Waveguides in Radio Astronomy

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Abstract: In terahertz radio astronomy, superconducting coplanar waveguide resonators have been commonly applied in Kinetic Inductance Detectors (KIDs) to measure the absorption of photon energy in the millimeter and submillimeter bands. Here, we present an analysis on the performance of superconducting niobium coplanar waveguides (CPWs). To compute the loss in a superconducting CPW, we have incorporated the complex conductivity developed by Matis and Bardeen based on the superconducting BCS theory, into the CPW loss equation. We have made a comparison between the loss in a CPW at room temperature with that below the critical temperature T_c of the superconductor. It can be observed that at frequencies below the gap frequency f_g, the low in the superconducting CPW is significantly lower than that in a normal CPW. Above f_g, however, the material loses its superconductivity and the loss in both temperatures becomes comparable. In our analysis, we have also shown that the loss decreases as the gap bewteen the strip and groundplane becomes wider. Hence, with careful design, the loss in a CPW can actuall y be minimized.

Keywords – Coplanar waveguides; Kinetic Inductance Detectors; complex conductivity; critical temperature; gap frequency

I. INTRODUCTION

The millimeter and submillimeter bands of the electromagnetic spectrum hold important spectral and spatial information in the field of astrophysics. For example, the study of cosmic microwave background (CMB) radiation which peaks in the frequency range of 100 GHz to 300 GHz provides an in depth understanding on the physics of the Big Bang theory and the formation of the early universe [1]. Besides, the cold material (10 K to 30 K) associated with the early stages of star and planet formation, as well as the earliest stages of galaxy formation, has its peak emission at the millimeter and submillimeter range as well [2].

In [3], P. K. Day *et al.* has developed the Kinetic Inductance Detector (KID) to measure the absorption of photon energy in the millimeter and submillimeter bands. KID is basically a superconducting pair-breaking detector which operates based on the detection of quasiparticles in a superconducting material at temperature T below the critical temperature T_c. When Cooper pairs absorbed radiation energy beyond the energy gap of a superconductor, they are broken into excited quasiparticles. The increase of quasiparticles changes the surface reactance of the material – a phenomenon known as the kinetic inductance effect. By incorporating properly designed capacitance, the variation of inductance can be measured using a resonator circuit. Such variation causes a shift in the resonant frequency f_0 which can then be measured based on the change of the amplitude and phase of the resonator circuit.

Superconducting coplanar waveguide (CPW) resonators have been commonly used in KIDs to couple the incoming millimeter and submillimeter signals and to detect the resonant frequency shift [4] – [7]. Here, we present an analysis on the performance of the superconducting niobium (Nb) coplanar waveguides typically used in a KID circuit. The objective of this paper is to investigate the loss efficiency of wave coupling to the CPW.

II. FORMULATION

Consider the coplanar waveguide (CPW) shown in Fig. 1 with the width of the groundplane g approaches infinity. The attenuation constant in a CPW α can be solved using the quasi-static approach in [7][8]

$$\alpha = \frac{R_s\sqrt{\varepsilon_{re}}}{480\,\pi K(k_1)K'(k_1)(1-k_1^{\,2})}\times \left\{\frac{1}{w}\left[\pi+\ln\left(\frac{8\pi w(1-k_1)}{t_s(1+k_1)}\right)\right]+\frac{1}{b}\left[\pi+\ln\left(\frac{8\pi b(1-k_1)}{t_s(1+k_1)}\right)\right]\right\} \quad (1)$$

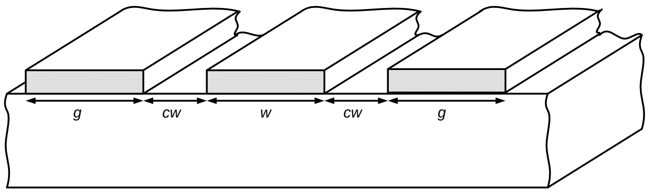

Figure 1. The cross section of a coplanar waveguide.

where R_s is the surface resistance of the strip, $b = w + cw$, $k_1 = w/(2b)$, w the width of the strip, t_s thickness of the strip, cw the gap between the width and the groundplane, ε_{re} is the effective dielectric constant, and $K(k_1)$ and $K'(k_1)$ are the complete elliptic integrals of the first kind and its complement, respectively. The effective dielectric constant can be found based on the conformal mapping method in [9]

$$\varepsilon_{re} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k_2)}{K'(k_2)} \frac{K'(k_1)}{K(k_1)} \qquad (2)$$

where ε_r is the dielectric constant of the substrate and

$$k_2 = \sinh(\pi w/2s)/\sinh(\pi b/2s) \qquad (3)$$

where s is the thickness of the substrate.

It is worthwhile noting that, the surface resistance R_S in (1) can be obtained by computing the surface impedance Z_S of the strip and groundplane and extracting the real part of Z_S. The surface impedance Z_S can be expressed in terms of the electrical properties of the conductor [10][11]

$$Z_s = \sqrt{\frac{\mu_w}{\varepsilon_w}} \qquad (4)$$

where μ_w and ε_w are the permeability and permittivity of the wall material, respectively. ε_w is complex and is given as [11]

$$\varepsilon_w = \varepsilon - j\frac{\sigma}{\omega} \qquad (5)$$

where σ is the conductivity of the conducting material, ε the permittivity of free space, and ω is the angular frequency. In order to estimate the loss of waves in millimetre and submillimeter wavelengths more accurately, a more evolved model than the conventional constant conductivity model used at microwave frequency is necessary. For a normal conductor, we have applied Drude's model for the frequency dependent conductivity σ [12]

$$\sigma = \frac{\sigma_n}{(1 + j\omega\tau)} \qquad (6)$$

where σ_n is the conventional constant conductivity [13] – [15] and τ the mean free time. For most conductors, the mean free time is in the range of 10^{-13} to 10^{-14} s [16].

Due to the existence of the energy gap $2\Delta(T)$ in a superconductor, however, σ is not only complex and frequency dependent, but temperature dependent as well. The equations for the complex conductivity have been developed by Mattis and Bardeen from the microscopic analysis of Bardeen-Cooper-Schrieffer (BCS) theory [17][18]

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar\omega}\int_{\Delta}^{\infty} \left[f(E) - f(E+\hbar\omega)\right] \frac{E^2 + \Delta^2 + \hbar\omega E}{(E^2-\Delta^2)^{1/2}\left[(E+\hbar\omega)^2 - \Delta^2\right]^{1/2}} dE + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta} \left[1 - 2f(E+\hbar\omega)\right] \frac{E^2 + \Delta^2 + \hbar\omega E}{(E^2-\Delta^2)^{1/2}\left[(E+\hbar\omega)^2 - \Delta^2\right]^{1/2}} dE \qquad (7)$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega,-\Delta}^{\Delta} \left[1 - 2f(E+\hbar\omega)\right] \frac{E^2 + \Delta^2 + \hbar\omega E}{(\Delta^2-E^2)^{1/2}\left[(E+\hbar\omega)^2 - \Delta^2\right]^{1/2}} dE \qquad (8)$$

where $\hbar$ is the reduced Planck's constant, σ_n the normal conductivity, and $\Delta = \Delta(T)$ the energy-gap parameter. The function,

$$f(E) = \frac{1}{1 + \exp\left(E/kT\right)} \qquad (9)$$

gives the Fermi-Dirac statistics and k is the Boltzmann's constant. The first integral in (7) describes the effect of the thermally excited quasiparticles. The second integral denotes the generation of quasiparticles by fields with frequencies f corresponding to energies above the gap energy. Thus, the second integral is zero for $\hbar\omega < 2\Delta$. Since σ_2 indicates the contribution due to the Cooper pairs, the lower integration limit in (8) becomes $-\Delta$ when $\hbar\omega > 2\Delta$. Δ depends on temperature and is obtained from the relation [19] [20]

$$\ln\left(\tilde{\Delta}\right) = -2\int_0^{\infty} \left(E^2 + \tilde{\Delta}^2\right)^{-1/2} \left\{1 + \exp\left[\left(\pi/\gamma_E \tilde{T}\right)\left(E^2 + \tilde{\Delta}^2\right)^{1/2}\right]\right\}^{-1} dE \qquad (10)$$

where $\tilde{\Delta} = \Delta(T)/\Delta(0)$, $\tilde{T} = T/T_c$, and $\gamma_E = 1.781$ is the Euler's constant.

III. Results and Discussion

The attenuation constant in a coplanar waveguide with a strip width w of 750 nm, strip thickness t_s of 300 nm, substrate thickness s of 250 nm, and separation distance between the strip and groundplane cw of 2 μm for a Nb CPW operating at both room temperature (T = 300 K) and temperature below the critical temperature T_c of Nb (T = 4.2 K) is shown in Fig. 2. Here, the critical temperature for Nb is given as T_c = 9.25 K [18]. As can be observed, the loss in the superconducting coplanar waveguide at frequency below the gap frequency f_g is significantly lower than that operating at room temperature. When the operating frequency f exceeds the gap frequency f_g of Nb which is approximately 700 GHz, the loss rises drastically. Indeed, at $f > f_g$, the loss in the CPW operating at 4.2 K is found to match closely that of a normal CPW. This implies that the CPW loses its superconductivity when the frequency f increases beyond the gap frequency f_g. Such phenomenon agrees very much with most superconducting planar devices, such as the superconducting microstrip lines in [19] and superconducting striplines in [20].

To understand this phenomenon, the superconducting complex conductivity of Nb at 4.2 K is depicted in Fig. 3 [11] [21]. As can be seen in Fig. 3, σ_1 which indicates the effect of quasiparticles is close to zero at frequencies below the gap frequency f_g. Above f_g, however, σ_1 increases gradually, approaching the value of the normal conductivity σ_n; whereas, σ_2 decreases slowly to zero. As explained in [18] [22] [23], the situation in a superconductor can be thought of as analogous to a semiconductor, with both having an energy gap at the Fermi surface. Cooper pairs absorbed energy higher than the gap energy at $f > f_g$ are capable of breaking into quasiparticles. Hence, when a superconductor is at finite temperatures below the critical temperature, T_c, thermal energy and incident radiation with energy level above the gap energy results in a large increase of quasiparticles due to Cooper pair breaking. As a result, niobium loses its superconducting behaviour at such high frequencies.

Next, we have also investigated the characteristics of a superconducting CPW by varying the distance between the strip and groundplanes cw, with the strip width w retained the same as that in Fig. 1. As depicted in Fig. 4, the loss in the superconducting CPW turns out to become lower when the distance cw becomes wider. Such result is important, since it strongly suggests that by properly designing the w to cw ratio, the loss in a coplanar waveguide can actually be minimized.

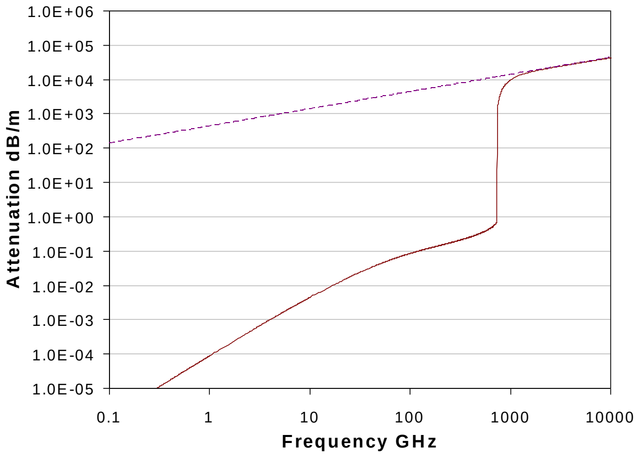

Figure 2. Attenuation in a coplanar waveguide at T = 300 K and 4.2 K.

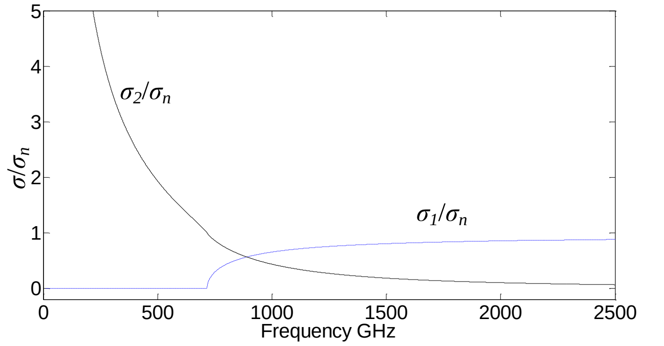

Figure 3. The normalized complex conductivity of Nb at 4.2 K.

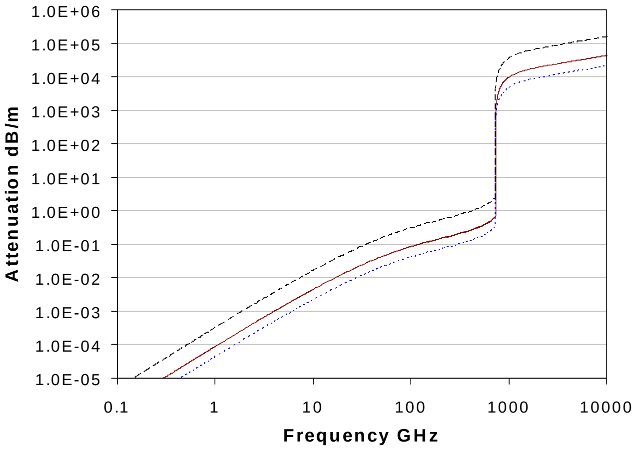

Figure 4. Attenuation in coplanar waveguide at cw = 0.2 µm (dashed line), cw = 2 µm (solid line), and cw = 20 µm (dotted line).

IV. CONCLUSIONS

As a conclusion, we have performed an analysis on a Nb coplanar waveguide (CPW). The loss in a superconducting CPW is computed by incorporating the complex conductivity formulated by Mattis and Bardeen into the loss equation. As shown in the results, when the superconducting CPW operates below the gap frequency f_g, the loss is considerably lower than when it is operating at room temperature. However, when the frequency exceeds f_g, Nb loses its superconductivity and the loss becomes comparable with that of a normal conductor. Analysis on the variation of the distance between the strip and groundplanes cw also shows that the loss of the superconducting CPW decreases as cw increases.

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