Analysis of Normal and Superconducting Coplanar Waveguides in Radio Astronomy

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Abstract: In terahertz radio astronomy, superconducting coplanar waveguide resonators have been commonly applied in Kinetic Inductance Detectors (KIDs) to measure the absorption of photon energy in the millimeter and submillimeter bands. Here, we present an analysis on the performance of superconducting niobium coplanar waveguides (CPWs). To compute the loss in a superconducting CPW, we have incorporated the complex conductivity developed by Mattis and Bardeen based on the superconducting BCS theory, into the CPW loss equation. We have made a comparison between the loss in a CPW at room temperature with that below the critical temperature Tc of the superconductor. It can be observed that at frequencies below the gap frequency f0, the low in the superconducting CPW is significantly lower than that in a normal CPW. Above f0, however, the material loses its superconductivity and the loss in both temperatures becomes comparable. In our analysis, we have also shown that the loss decreases as the gap between the strip and groundplane becomes wider. Hence, with careful design, the loss in a CPW can actually be minimized.

Keywords – Coplanar waveguides; Kinetic Inductance Detectors; complex conductivity; critical temperature; gap frequency

I. INTRODUCTION

The millimeter and submillimeter bands of the electromagnetic spectrum hold important spectral and spatial information in the field of astrophysics. For example, the study of cosmic microwave background (CMB) radiation which peaks in the frequency range of 100 GHz to 300 GHz provides an in depth understanding on the physics of the Big Bang theory and the formation of the early universe [1]. Besides, the cold material (10 K to 30 K) associated with the early stages of star and planet formation, as well as the earliest stages of galaxy formation, has its peak emission at the millimeter and submillimeter range as well [2].

In [3], P. K. Day et al. has developed the Kinetic Inductance Detector (KID) to measure the absorption of photon energy in the millimeter and submillimeter bands. KID is basically a superconducting pair-breaking detector which operates based on the detection of quasiparticles in a superconducting material at temperature \( T \) below the critical temperature \( T_c \). When Cooper pairs absorbed radiation energy beyond the energy gap of a superconductor, they are broken into excited quasiparticles. The increase of quasiparticles changes the surface reactance of the material – a phenomenon known as the kinetic inductance effect. By incorporating properly designed capacitance, the variation of inductance can be measured using a resonator circuit. Such variation causes a shift in the resonant frequency \( f_0 \) which can then be measured based on the change of the amplitude and phase of the resonant circuit.

Superconducting coplanar waveguide (CPW) resonators have been commonly used in KIDs to couple the incoming millimeter and submillimeter signals and to detect the resonant frequency shift [4] – [7]. Here, we present an analysis on the performance of the superconducting niobium (Nb) coplanar waveguides typically used in a KID circuit. The objective of this paper is to investigate the loss efficiency of wave coupling to the CPW.

II. FORMULATION

Consider the coplanar waveguide (CPW) shown in Fig. 1 with the width of the groundplane \( g \) approaches infinity. The attenuation constant in a CPW \( \alpha \) can be solved using the quasi-static approach in [7][8]

\[
\alpha = \frac{R_s \sqrt{R_w}}{480 \pi K (k_1) K' (k_1) (1 - k_1)} \times \left\{ \frac{1}{\pi} \ln \left( \frac{8 \pi \nu (1 - k)}{\nu (1 + k)} \right) + \frac{1}{b} \ln \left( \frac{8 \pi b (1 - k)}{\nu (1 + k)} \right) \right\}
\]

(1)
Due to the existence of the energy gap $2\Delta(T)$ in a superconductor, however, $\sigma$ is not only complex and frequency dependent, but temperature dependent as well. The equations for the complex conductivity have been developed by Mattis and Bardeen from the microscopic analysis of Bardeen-Cooper-Schrieffer (BCS) theory [17][18]

$$\frac{\sigma_2}{\sigma_n} = \frac{2}{\hbar \omega} \int (E - f(E + \hbar \omega) - \frac{E^2 + \Delta^2 + \hbar \omega E}{(E^2 - \Delta^2 + \hbar \omega E)^{1/2}} - \frac{E^2 + \Delta^2 + \hbar \omega E}{(E^2 - \Delta^2 - \hbar \omega E)^{1/2}}) \frac{dE}{\hbar \omega}$$

$$+ \frac{1}{\hbar \omega} \int \left[ \frac{2 f(E + \hbar \omega) - (E^2 + \Delta^2 + \hbar \omega E) - (E^2 + \Delta^2 - \hbar \omega E)}{(E^2 - \Delta^2 + \hbar \omega E)^{1/2} - \Delta^2 - \hbar \omega E)^{1/2}} \frac{dE}{\hbar \omega} \right]$$

(7)

where $\hbar$ is the reduced Planck’s constant, $\sigma_n$ the normal conductivity, and $\Delta = \Delta(T)$ the energy-gap parameter. The function,

$$f(E) = \frac{1}{1 + \exp \left( \frac{\Delta}{k T} \right)}$$

(9)

gives the Fermi-Dirac statistics and $k$ is the Boltzmann’s constant. The first integral in (7) describes the effect of the thermally excited quasiparticles. The second integral denotes the generation of quasiparticles by fields with frequencies $f$ corresponding to energies above the gap energy. Thus, the second integral is zero for $\hbar \omega < 2\Delta$. Since $\sigma_2$ indicates the contribution due to the Cooper pairs, the lower integration limit in (8) becomes $-\Delta$ when $\hbar \omega > 2\Delta$. $\Delta$ depends on temperature and is obtained from the relation [19][20]

$$\ln \left( \tilde{\Delta} \right) = -2\gamma_E \left( E + \tilde{\Delta} \right)^{1/2} \left[ 1 + \exp \left( \frac{\sigma / \gamma_E T_0 \left( E + \tilde{\Delta} \right)^{1/2}}{1 + \exp \left( \sigma / \gamma_E T_0 \left( E + \tilde{\Delta} \right)^{1/2} \right) \frac{dE}{\hbar \omega}} \right) \right]$$

(10)

where $\tilde{\Delta} = \Delta(T)/\Delta(0)$, $\tilde{T} = T/T_c$, and $\gamma_E = 1.781$ is the Euler’s constant.

III. RESULTS AND DISCUSSION

The attenuation constant in a coplanar waveguide with a strip width $w$ of 750 nm, strip thickness $t_s$ of 300 nm, substrate thickness $s$ of 250 nm, and separation distance between the strip and groundplane $cw$ of 2 $\mu$m for a Nb CPW operating at both room temperature ($T = 300$ K) and temperature below the critical temperature $T_c$ of Nb ($T = 4.2$ K) is shown in Fig. 2. Here, the critical temperature for Nb is given as $T_c = 9.25$ K [18]. As can be observed, the loss in the superconducting coplanar waveguide at frequency below the gap frequency $f_g$ is significantly lower than that operating at room temperature. When the operating frequency $f$ exceeds the gap frequency $f_g$ of Nb,

$$\sigma = \frac{\sigma_0}{(1 + j \omega \tau)}$$

(10)

where $\sigma_0$ is the conventional constant conductivity [13],[15] and $\tau$ the mean free time. For most conductors, the mean free time is in the range of $10^{-13}$ to $10^{-14}$ s [16].

Figure 1. The cross section of a coplanar waveguide.
which is approximately 700 GHz, the loss rises drastically. Indeed, at \( f > f_c \), the loss in the CPW operating at 4.2 K is found to match closely that of a normal CPW. This implies that the CPW loses its superconductivity when the frequency \( f \) increases beyond the gap frequency \( f_c \). Such phenomenon agrees very much with most superconducting planar devices, such as the superconducting microstrip lines in [19] and superconducting striplines in [20].

To understand this phenomenon, the superconducting complex conductivity of Nb at 4.2 K is depicted in Fig. 3 [11] [21]. As can be seen in Fig. 3, \( \sigma_I \) which indicates the effect of quasiparticles is close to zero at frequencies below the gap frequency \( f_c \). Above \( f_c \), however, \( \sigma_I \) increases gradually, approaching the value of the normal conductivity \( \sigma_n \); whereas, \( \sigma_2 \) decreases slowly to zero. As explained in [18] [22] [23], the situation in a superconductor can be thought of as analogous to a semiconductor, with both having an energy gap at the Fermi surface. Cooper pairs absorbed energy higher than the gap energy at \( f > f_c \) are capable of breaking into quasiparticles. Hence, when a superconductor is at finite temperatures below the critical temperature, \( T_c \), thermal energy and incident radiation with energy level above the gap energy results in a large increase of quasiparticles due to Cooper pair breaking. As a result, niobium loses its superconducting behaviour at such high frequencies.

Next, we have also investigated the characteristics of a superconducting CPW by varying the distance between the strip and groundplanes \( cw \), with the strip width \( w \) retained the same as that in Fig. 1. As depicted in Fig. 4, the loss in the superconducting CPW turns out to become lower when the distance \( cw \) becomes wider. Such result is important, since it strongly suggests that by properly designing the \( w \) to \( cw \) ratio, the loss in a coplanar waveguide can actually be minimized.

IV. CONCLUSIONS

As a conclusion, we have performed an analysis on a Nb coplanar waveguide (CPW). The loss in a superconducting CPW is computed by incorporating the complex conductivity formulated by Mattis and Bardeen into the loss equation. As shown in the results, when the superconducting CPW operates below the gap frequency \( f_c \), the loss is considerably lower than when it is operating at room temperature. However, when the frequency exceeds \( f_c \), Nb loses its superconductivity and the loss becomes comparable with that of a normal conductor. Analysis on the variation of the distance between the strip and groundplanes \( cw \) also shows that the loss of the superconducting CPW decreases as \( cw \) increases.

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REFERENCES


